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Mark Scheme (Results)

June 2011

GCE Core Mathematics C4 (6666) Paper 1

June 2011
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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - ☐ The second mark is dependent on gaining the first mark

June 2011
Core Mathematics C4 6666
Mark Scheme

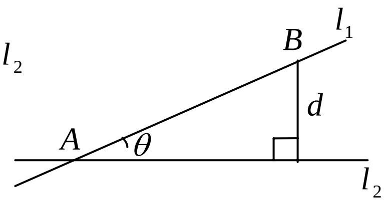
Question Number	Scheme		Marks	
1.	$9x^2 = A(x-1)(2x+1)+B(2x+1)+C(x-1)^2$		B1	
	$x \to 1$ $\quad 9 = 3B \Rightarrow B = 3$		M1	
	$x \to -\frac{1}{2}$ $\quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$	Any two of *A*, *B*, *C*	A1	
	x^2 terms $\quad 9 = 2A + C \Rightarrow A = 4$	All three correct	A1	**(4)** **[4]**
	Alternatives for finding A.			
	x terms $\quad 0 = -A + 2B - 2C \Rightarrow A = 4$ Constant terms $\quad 0 = -A + B + C \Rightarrow A = 4$			

Question Number	Scheme		Marks	
2.	$\mathrm{f}(x) = (\ \dots\ +\ \dots)^{-\frac{1}{2}}$		M1	
	$= 9^{-\frac{1}{2}}(\ \dots\ +\ \dots)^{\dots}$	3^{-1}, $\frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$	B1	
	$(1+kx^2)^n = 1 + nkx^2 + \dots$	n not a natural number, $k \neq 1$	M1	
	$(1+kx^2)^{-\frac{1}{2}} = \dots + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx^2)^2$	ft their $k \neq 1$	A1 ft	
	$\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1 - \frac{2}{9}x^2 + \frac{2}{27}x^4$		A1	
	$\mathrm{f}(x) = \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$		A1	**(6)**
				[6]

Question Number	Scheme		Marks	
3.	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$	or equivalent	M1 A1	
	At $h = 0.1$, $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$	M1 A1	**(4)**
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a)	M1	
	At $h = 0.1$, $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031	A1	**(2)**
				[6]

Question Number	Scheme	Marks
4.	(a) 0.0333, 1.3596 awrt 0.0333, 1.3596	B1 B1 **(2)**
	(b) $\text{Area}(R) \approx \frac{1}{2}\times\frac{\sqrt{2}}{4}[\ ...]$	B1
	$\approx\ ...\ \left[0+2\left(0.0333+0.3240+1.3596\right)+3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)**
	(c) $u = x^2+2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$	B1
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln\left(x^2+2\right)\mathrm{d}x$	B1
	$\int x^3 \ln\left(x^2+2\right)\mathrm{d}x = \int x^2 \ln\left(x^2+2\right)x\,\mathrm{d}x = \int (u-2)(\ln u)\tfrac{1}{2}\mathrm{d}u$	M1
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,\mathrm{d}u$ ✱ cso	A1 **(4)**
	(d) $\int (u-2)\ln u\,\mathrm{d}u = \left(\frac{u^2}{2}-2u\right)\ln u - \int\left(\frac{u^2}{2}-2u\right)\frac{1}{u}\mathrm{d}u$	M1 A1
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \int\left(\frac{u}{2}-2\right)\mathrm{d}u$	
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\quad(+C)$	M1 A1
	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\right]_2^4$	
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - \left((2-4)\ln 2 - 1 + 4\right)\right]$	M1
	$= \tfrac{1}{2}(2\ln 2 + 1)$ $\qquad \ln 2 + \tfrac{1}{2}$	A1 **(6)** **[15]**

Question Number	Scheme		Marks	
5.	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ \ldots$		B1	
	$\ldots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$		M1 A1	
	At $x = 2$, $\quad \ln y = 2(2)\ln 2$		M1	
	leading to $\quad y = 16$	Accept $y = \mathrm{e}^{4\ln 2}$	A1	
	At $(2, 16)$ $\quad \frac{1}{16}\frac{\mathrm{d}y}{\mathrm{d}x} = 2\ln 2 + 2$		M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 16(2 + 2\ln 2)$		A1	**(7)**
				[7]
	Alternative			
	$y = \mathrm{e}^{2x\ln x}$		B1	
	$\frac{\mathrm{d}}{\mathrm{d}x}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$		M1 A1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)\mathrm{e}^{2x\ln x}$		M1 A1	
	At $x = 2$, $\quad \frac{\mathrm{d}y}{\mathrm{d}x} = (2\ln 2 + 2)\mathrm{e}^{4\ln 2}$		M1	
	$= 16(2 + 2\ln 2)$		A1	**(7)**

Question Number	Scheme		Marks	
6.	(a) **i**: $6-\lambda=-5+2\mu$			
	j: $-3+2\lambda=15-3\mu$	Any two equations	M1	
	leading to $\lambda=3,\ \mu=4$		M1 A1	
	$\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}-5\\15\\3\end{pmatrix}+4\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$		M1 A1	
	k: LHS $=-2+3(3)=7$**,** RHS $=3+4(1)=7$		B1	**(6)**
	(As LHS = RHS, lines intersect)			
	Alternatively for B1, showing that $\lambda=3$ and $\mu=4$ both give $\begin{pmatrix}3\\3\\7\end{pmatrix}$			
	(b) $\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix}=-2-6+3=\surd14\surd14\cos\theta$ $(\theta\approx110.92°)$		M1 A1	
	Acute angle is $69.1°$	awrt 69.1	A1	**(3)**
	(c) $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+1\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}5\\-1\\1\end{pmatrix}$ $(\Rightarrow B \text{ lies on } l_1)$		B1	**(1)**
	(d) Let d be shortest distance from B to l_2 			
	$\overrightarrow{AB}=\begin{pmatrix}5\\-1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$		M1	
	$\lvert\overrightarrow{AB}\rvert=\surd\left(2^2+(-4)^2+(-6)^2\right)=\surd56$	awrt 7.5	A1	
	$\dfrac{d}{\surd56}=\sin\theta$		M1	
	$d=\surd56\sin69.1°\approx6.99$	awrt 6.99	A1	**(4)**
				[14]

Question Number	Scheme	Marks
7.	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$	M1
	$\theta = \frac{\pi}{3}$ awrt 1.05	A1 **(2)**
	(b) $\frac{dx}{d\theta} = \sec^2\theta$, $\frac{dy}{d\theta} = \cos\theta$	
	$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta}$ $\left(= \cos^3\theta\right)$	M1 A1
	At P, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$ Can be implied	A1
	Using $mm' = -1$, $m' = -8$	M1
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	M1
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$ 1.0625	A1 **(6)**
	(c) $\int y^2\,dx = \int y^2 \frac{dx}{d\theta}\,d\theta = \int \sin^2\theta \sec^2\theta\,d\theta$	M1 A1
	$= \int \tan^2\theta\,d\theta$	A1
	$= \int \left(\sec^2\theta - 1\right)d\theta$	M1
	$= \tan\theta - \theta$ $(+C)$	A1
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\,dx = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - (0 - 0)\right]$	M1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2$ $\left(p = 1, q = -\frac{1}{3}\right)$	A1 **(7)**
		[15]

Question Number	Scheme	Marks
8.	(a) $\int (4y+3)^{-\frac{1}{2}}\,dx = \dfrac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1 **(2)**
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
	(b) $\int \dfrac{1}{\sqrt{(4y+3)}}\,dy = \int \dfrac{1}{x^2}\,dx$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,dy = \int x^{-2}\,dx$	
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \dfrac{1}{2}(4\times1.5+3)^{\frac{1}{2}} = -\dfrac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \dfrac{2}{x}$	M1
	$y = \dfrac{1}{4}\left(2 - \dfrac{2}{x}\right)^2 - \dfrac{3}{4}$ $\quad$ or equivalent	A1 **(6)**
		[8]

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