June 2009
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme		Marks
1. (a)	Iterative formula: $x_{n+1} = \dfrac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \dfrac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$ $x_2 = 2.371581451...$	Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372	A1
	$x_3 = 2.355593575...$ $x_4 = 2.360436923...$	Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36	A1 **cso**
			[3]
(b)	Let $\text{f}(x) = -x^3 + 2x^2 + 2 = 0$		
	$\text{f}(2.3585) = 0.00583577...$ $\text{f}(2.3595) = -0.00142286...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
		any one value awrt 1 sf or truncated 1 sf	dM1
	Sign change (and $\text{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
			[3]
			6 marks

See appendix!

At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".

Question Number	Scheme		Marks
2. (a)	$\cos^2\theta + \sin^2\theta = 1 \quad (\div \cos^2\theta)$		
	$\underline{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}}$	Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation.	M1
	$1 + \tan^2\theta = \sec^2\theta$		
	$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG**	Complete proof. No errors seen. (see alternatives in appendix!)	A1 **cso** **[2]**
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2, \quad (\text{eqn } *) \quad 0 \le \theta < 360^\circ$		
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$		
	$\underline{3\sec^2\theta + 4\sec\theta - 4} = 0$	Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic. (See rules for factorising quadratics)	M1
	$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$		
	$\dfrac{1}{\cos\theta} = -2$ or $\dfrac{1}{\cos\theta} = \dfrac{2}{3}$		
	$\underline{\cos\theta = -\frac{1}{2}}$; or $\cos\theta = \frac{3}{2}$	$\underline{\cos\theta = -\frac{1}{2}}$	A1;
	$\alpha = 120^\circ$ or $\alpha =$ no solutions		
	$\theta_1 = \underline{120^\circ}$	$\underline{120^\circ}$	A1
	$\theta_2 = 240^\circ$	$\underline{240^\circ}$ or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = ...$	B1 $\sqrt{}$
	$\theta = \{120^\circ, 240^\circ\}$	Note the final A1 mark has been changed to a B1 mark.	**[6]**
			8 marks

Note: Please refer to the appendix if candidate offers any extra solutions.

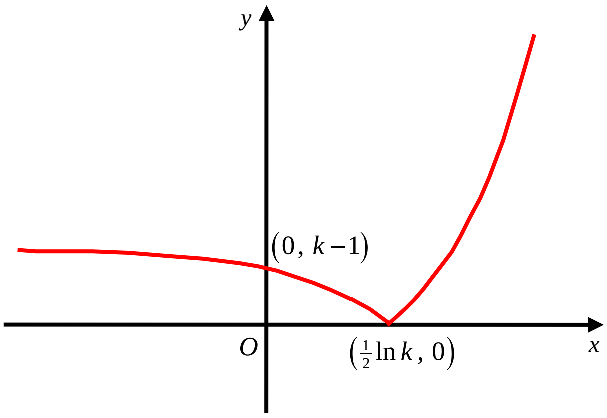
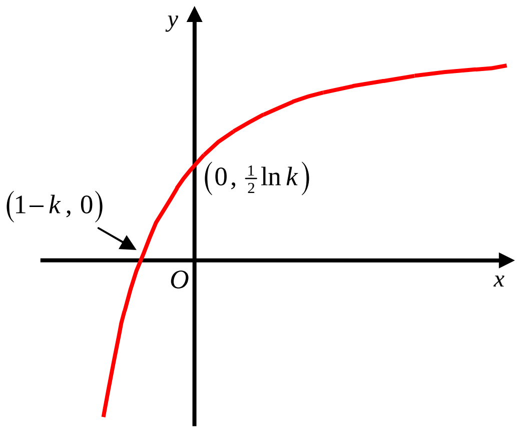
Question Number	Scheme		Marks
3.	$P = 80\,e^{\frac{t}{5}}$		
(a)	$t = 0 \Rightarrow P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1 **[1]**
(b)	$P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{\frac{t}{5}}$	Substitutes $P = 1000$ and rearranges equation to make $e^{\frac{t}{5}}$ the subject.	M1
	$\therefore\ t = 5\ln\left(\frac{1000}{80}\right)$		
	$t = 12.6286...$	awrt 12.6 or 13 years	A1 **[2]**
(c)	$\frac{dP}{dt} = 16e^{\frac{t}{5}}$	$ke^{\frac{1}{5}t}$ and $k \neq 80$.	M1
		$16e^{\frac{1}{5}t}$	A1 **[2]**
(d)	$50 = 16e^{\frac{t}{5}}$		
	$\therefore\ t = 5\ln\left(\frac{50}{16}\right)$ $\{= 5.6797...\}$	Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$.	M1
	$P = 80\,e^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\,e^{\frac{1}{5}(5.6797...)}$	Substitutes their value of t back into the equation for P.	dM1
	$P = \frac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1 **[3]**
			8 marks

Note $t = 12$
or $t =$ awrt 12.6 $\Rightarrow t = 12$ will score A0.

Question Number	Scheme		Marks
4. (i) (a)	$y = x^2\cos 3x$		
	Apply product rule: $\left\{\begin{array}{ll} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{array}\right\}$		
	$\frac{dy}{dx} = 2x\cos 3x - 3x^2\sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x\cos 3x \pm \beta x^2\sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos\alpha x^2$ or $\sin\beta x^3$.	A1
			[3]
(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2+1}$	$\ln(x^2+1) \to \frac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \to \frac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{\begin{array}{ll} u = \ln(x^2+1) & v = x^2+1 \\ \frac{du}{dx} = \frac{2x}{x^2+1} & \frac{dv}{dx} = 2x \end{array}\right\}$		
	$\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	Applying $\frac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			[4]
	$\left\{\frac{dy}{dx} = \frac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}\right\}$	{Ignore subsequent working.}	

Question Number	Scheme		Marks
4. (ii)	$y = \sqrt{4x+1},\ \ x > -\frac{1}{4}$		
	At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$	At P, $y = \underline{\sqrt{9}}$ or $\underline{3}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	$\pm k(4x+1)^{-\frac{1}{2}}$ $2(4x+1)^{-\frac{1}{2}}$	M1* A1 **aef**
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\left(4(2)+1\right)^{\frac{1}{2}}}$	Substituting $x = 2$ into an equation involving $\frac{dy}{dx}$;	M1
	Hence m(**T**) = $\frac{2}{3}$		
	Either **T**: $y - 3 = \frac{2}{3}(x-2)$; or $y = \frac{2}{3}x + c$ and $3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$;	$y - y_1 = m(x-2)$ or $y - y_1 = m(x - \text{their stated } x)$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either **T**: $3y - 9 = 2(x-2)$; **T**: $3y - 9 = 2x - 4$ **T**: $\underline{2x - 3y + 5 = 0}$ or **T**: $y = \frac{2}{3}x + \frac{5}{3}$ **T**: $3y = 2x + 5$ **T**: $\underline{2x - 3y + 5 = 0}$	$\underline{2x - 3y + 5 = 0}$	A1 **[6]**
			13 marks

Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers.

Question Number	Scheme		Marks
5. (a)		Curve retains shape when $x > \frac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$	B1
		$(0,\ k-1)$ and $(\frac{1}{2}\ln k,\ 0)$ marked in the correct positions.	B1
			[3]
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k,\ 0)$ and $(0,\ \frac{1}{2}\ln k)$	B1
			[2]
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$ or $\underline{f\ > -k}$ or $\underline{\text{Range} > -k}$.	B1
			[1]
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$	$\underline{\frac{1}{2}\ln(x+k)}$	A1 cao
			[3]
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer (see appendix)	B1 $\sqrt{}$
			[1]
			10 marks

Question Number	Scheme		Marks
6. (a)	$A=B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A=B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives		
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	Complete proof, with a link between LHS and RHS. No errors seen.	A1 **AG** **[2]**
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1
	$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$		
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$		
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 **AG** **[3]**
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$		
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$		
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$		
	$R = \sqrt{3^2 + 4^2}\ ;= \sqrt{25} = 5$	$R = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	$\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$ awrt 36.87	M1 A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$		**[3]**

Question Number	Scheme		Marks
6. (d)	$3\sin 2x + 4\cos 2x = 2$		
	$5\cos(2x - 36.87) = 2$		
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1
	$(2x - 36.87) = 66.42182...^{\circ}$	awrt 66	A1
	$(2x - 36.87) = 360 - 66.42182...^{\circ}$		
	Hence, $x = 51.64591...^{\circ}, \; 165.22409...^{\circ}$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3 Both of awrt 51.6 or awrt 165.2	A1 A1 **[4]**
			12 marks

If there are any EXTRA solutions inside the range $0 \le x < 180^{\circ}$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^{\circ}$.

Question Number	Scheme		Marks
7.	$\mathrm{f}(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$		
(a)	$\mathrm{f}(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	An attempt to combine to one fraction Correct result of combining all three fractions	M1 A1
	$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$		
	$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$	Simplifies to give the correct numerator. Ignore omission of denominator	A1
	$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	An attempt to factorise the numerator. See rules for factorising quadratics.	dM1
	$= \dfrac{(x-3)}{(x-2)}$	Correct result	A1 cso **AG** **[5]**
(b)	$\mathrm{g}(x) = \dfrac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$		
	Apply quotient rule: $\left\{ \begin{array}{ll} u = e^x - 3 & v = e^x - 2 \\ \dfrac{du}{dx} = e^x & \dfrac{dv}{dx} = e^x \end{array} \right\}$		
	$\mathrm{g}'(x) = \dfrac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$ Correct differentiation	M1 A1
	$= \dfrac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$		
	$= \dfrac{e^x}{(e^x - 2)^2}$	Correct result	A1 **AG** **cso** **[3]**

Question Number	Scheme		Marks
7. (c)	$g'(x) = 1 \Rightarrow \dfrac{e^x}{(e^x - 2)^2} = 1$		
	$e^x = (e^x - 2)^2$	Puts their differentiated numerator equal to their denominator.	M1
	$e^x = e^{2x} - 2e^x - 2e^x + 4$		
	$\underline{e^{2x} - 5e^x + 4} = 0$	$\underline{e^{2x} - 5e^x + 4}$	A1
	$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x	M1
	$e^x = 4$ or $e^x = 1$		
	$x = \ln 4$ or $x = 0$	both $x = 0,\ \ln 4$	A1 **[4]**
			12 marks

Question Number	Scheme		Marks
8. (a)	$\sin 2x = \underline{2\sin x \cos x}$	$\underline{2\sin x\cos x}$	B1 aef **[1]**
(b)	$\operatorname{cosec} x - 8\cos x = 0\,,\quad 0 < x < \pi$		
	$\frac{1}{\sin x} - 8\cos x = 0$	Using $\operatorname{cosec} x = \frac{1}{\sin x}$	M1
	$\frac{1}{\sin x} = 8\cos x$		
	$1 = 8\sin x \cos x$		
	$1 = 4(2\sin x\cos x)$		
	$1 = 4\sin 2x$		
	$\underline{\sin 2x = \frac{1}{4}}$	$\sin 2x = k$, where $-1 < k < 1$ $\underline{\sin 2x = \frac{1}{4}}$	M1 A1
	Radians $2x = \{0.25268..., 2.88891...\}$ Degrees $2x = \{14.4775..., 165.5225...\}$		
	Radians $x = \{0.12634..., 1.44445...\}$ Degrees $x = \{7.23875..., 82.76124...\}$	Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π. Both $\underline{0.13}$ and $\underline{1.44}$	A1 A1 **cao** **[5]**
			6 marks

Solutions for the final two A marks must be given in x only.

If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.