Question Number	Scheme	Marks
1. (*a*)		

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	e^2
or y	1	1.08329…	1.37713…	2.05443…	3.59664…	7.38906…

B1 (1)

(*b*) Area $\approx \frac{1}{2} \times 0.4 ; \times \left[e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28} \right) + e^2 \right]$ — B1; M1

$= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922}$ (4sf) — A1 (3)

(4 marks)

2. (a)

$$\left\{\begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$$

$\int x e^x \, dx = x e^x - \int e^x .1 \, dx$ — M1 A1

$= x e^x - \int e^x \, dx$

$= x e^x - e^x + c$ — A1 (3)

(*b*)

$$\left\{\begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$$

$\int x^2 e^x \, dx = x^2 e^x - \int e^x .2x \, dx$ — M1 A1

$= x^2 e^x - 2 \int x e^x \, dx$

$= x^2 e^x - 2\left(x e^x - e^x \right) + c$ — A1 (3)

(6 marks)

Question Number	Scheme	Marks
3. (*a*)	From question, $\frac{dA}{dt} = 0.032$	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} = \right\} 2\pi x$	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = 0.032 \frac{1}{2\pi x}; \left\{= \frac{0.016}{\pi x}\right\}$	M1
	When $x = 2\,\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$	
	Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s^{-1})	A1 cso (4)
(*b*)	$V = \underline{\pi x^2 (5x)} = \underline{5\pi x^3}$	B1
	$\frac{dV}{dx} = 15\pi x^2$	B1 ft
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2 . \left(\frac{0.016}{\pi x}\right); = 0.24x$	M1
	When $x = 2\,\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ ($\text{cm}^3\,\text{s}^{-1}$)	A1 (4)
		(8 marks)
4. (*a*)	$3x^2 - y^2 + xy = 4$ (eqn $*$)	
	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \times \right\}$ $\underline{6x - 2y\frac{dy}{dx}} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) = \underline{0}$	M1 B1 A1
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	M1
	giving $-18x - 3y = 8x - 16y$	
	giving $13y = 26x$	M1
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	A1 cso (6)
(*b*)	At *P* & *Q*, $y = 2x$. Substituting into eqn $*$	
	gives $3x^2 - (2x)^2 + x(2x) = 4$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	A1
	$y = 2x \Rightarrow y = \pm 4$, hence coordinates are $\underline{(2, 4)}$ and $\underline{(-2, -4)}$	A1 (3)
		(9 marks)

Question Number	Scheme	Marks
5. (*a*)	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{4^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$	B1
	$= \frac{1}{2}\left[\underline{1+(-\tfrac{1}{2})(**x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(**x)^2+...}\right]$ **with** $** \neq 1$	M1; A1 ft
	$= \frac{1}{2}\left[\underline{1+(-\tfrac{1}{2})(-\tfrac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\tfrac{3x}{4})^2+...}\right]$	
	$= \frac{1}{2}\left[1+\frac{3}{8}x;+\frac{27}{128}x^2+...\right]$	A1 A1 (5)
(*b*)	$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+...\right)$	M1
	$= \frac{1}{2}x+\frac{3}{16}x^2+.....$ $+ 4+\frac{3}{2}x+\frac{27}{32}x^2+.....$	M1
	$= 4+2x;+\frac{33}{32}x^2+...$	A1; A1 (4)
		(9 marks)

Question Number	Scheme	Marks
6. (*a*)	Lines meet where: $\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$	
	Any two of **i**: $-9+2\lambda=3+3\mu$ (1); **j**: $\lambda=1-\mu$ (2); **k**: $10-\lambda=17+5\mu$ (3)	M1
	(1) – 2**(2)** gives: $-9=1+5\mu \Rightarrow \mu=-2$	M1
	(2) gives: $\lambda=1--2=3$	A1
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	M1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	A1
	Either check **k:** $\lambda=3$: LHS $=10-\lambda=10-3=7$ $\mu=-2$: RHS $=17+5\mu=17-10=7$	B1 (6)
(*b*)	$\mathbf{d}_1=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$	
	As $\mathbf{d}_1\bullet\mathbf{d}_2=\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix}=\underline{(2\times3)+(1\times-1)+(-1\times5)}=0$	M1 A1 (2)
	Then l_1 is perpendicular to l_2.	
(*c*)	Equating **i** ; $-9+2\lambda=5 \Rightarrow \lambda=7$	
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+7\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}5\\7\\3\end{pmatrix}$ $(=\overrightarrow{OA}.$ Hence the point A lies on l_1.)	B1 (1)
(*d*)	Let $\overrightarrow{OX}=-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ be point of intersection	
	$\overrightarrow{AX}=\overrightarrow{OX}-\overrightarrow{OA}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}}=\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	M1 ft
	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AX}$	

Question Number	Scheme	Marks
	$\overrightarrow{OB} = \begin{pmatrix}5\\7\\3\end{pmatrix} + 2\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	M1 ft
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$	A1
		(12 marks)
7. (*a*)	$\dfrac{2}{4-y^2} \equiv \dfrac{2}{(2-y)(2+y)} \equiv \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$ so $2 \equiv A(2+y) + B(2-y)$	M1
	Let $y=-2$, $2 = B\ 4 \Rightarrow B = \frac{1}{2}$, Let $y=2$, $2 = A\ 4 \Rightarrow A = \frac{1}{2}$	M1
	giving $\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$	A1 cao (3)
(*b*)	$\displaystyle\int \frac{2}{4-y^2}\,\mathrm{d}y = \int \frac{1}{\cot x}\,\mathrm{d}x$	B1
	$\displaystyle\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,\mathrm{d}y = \int \tan x\,\mathrm{d}x$	
	$\therefore\ -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x)\ +\ c$	B1 M1 A1 ft
	$y=0,\ x=\frac{\pi}{3}\ \Rightarrow\ -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln \frac{1}{\cos\frac{\pi}{3}} + c$	M1
	$0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}$	
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$	
	$\dfrac{1}{2}\ln\left(\dfrac{2+y}{2-y}\right) = \ln\left(\dfrac{\sec x}{2}\right)$	M1
	$\ln\left(\dfrac{2+y}{2-y}\right) = 2\ln\left(\dfrac{\sec x}{2}\right)$	
	$\ln\left(\dfrac{2+y}{2-y}\right) = \ln\left(\dfrac{\sec x}{2}\right)^2$	M1
	$\dfrac{2+y}{2-y} = \dfrac{\sec^2 x}{4}$	
	Hence, $\underline{\sec^2 x = \dfrac{8+4y}{2-y}}$	A1 (8)
		(11 marks)

Question Number	Scheme	Marks
8. (*a*)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
(*b*)	$x = 8\cos t, \quad y = 4\sin 2t$	
	$\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t$	M1 A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos \frac{2\pi}{3}}{-8\sin \frac{\pi}{3}}}$	M1
	$\left\{ = \frac{8\ -\frac{1}{2}}{-8\ \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$	
	Hence m(**N**) = $-\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	M1
	N: $y - 2\sqrt{3} = -\sqrt{3}\ x - 4$	M1
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ (*)	A1 cso (6)
(*c*)	$A = \int_0^4 y\,dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.\ -8\sin t\ \ dt$	M1 A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\ dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\ \ 2\sin t\cos t\ \ .\sin t\ dt$	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\ dt$	
	$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\ dt$ (*)	A1 (4)
(*d*)	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	M1 A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	M1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	A1 (4)
		(16 marks)